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Issues in Determining α_s from Hadronic τ Decay and Electroproduction Data

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Abstract

We discuss some key issues associated with duality-violating and non-perturbative OPE contributions to the theoretical representations of light quark current-current two-point functions and relevant to precision determinations of α_s from hadronic τ decay and electroproduction cross-section data. We demonstrate that analyses with an explicit representation of duality-violating effects are required to bring theoretical errors in such extractions under control, motivating the accompanying paper in these proceedings, which presents the results of such an analysis.

Keywords: α_s, τ decay, duality violation

1. Introduction

Determinations of α_s based on hadronic τ decay or electroproduction cross-section data rely on the fact that, for any $s_0 > 0$ and any analytic $w(s)$, appropriate combinations, $\Pi(s)$, of the $J = 0, 1$ scalar components, $\Pi^{(J)}(s)$, of the relevant current-current two-point functions satisfy the finite energy sum rule (FESR) relation

$$\int_0^{s_0} ds\, w(s)\, \rho(s) \;=\; \frac{-1}{2\pi i}\oint_{|s|=s_0} ds\, w(s)\, \Pi(s)\,, \qquad (1)$$

with $\rho(s) = \frac{1}{\pi}\, Im\, \Pi(s)$ the spectral function of Π. If the current in question is either of the flavor ud vector (V) or axial vector (A) currents, the LHS of Eq. (1) can be determined experimentally, for $s_0 \leq m_\tau^2$, using hadronic τ decay data [1, 2]. For the electromagnetic (EM) current case, the LHS can similarly be determined using inclusive electroproduction cross-sections. A determination of α_s is then possible because, for large enough s_0, the OPE representation of the RHS is dominated by its dimension $D = 0$ perturbative contribution. Additional contributions to the RHS result from higher D, non-perturbative (NP) terms in the OPE representation of Π, as well as from any deviations ("duality violations" or DVs) between this representation and Π itself. The precision with which α_s can be determined is affected by the accuracy with which these small, but not generally negligible, contributions can be estimated.

With $Q^2 = -s$, the OPE representation of Π becomes

$$\left[\Pi(Q^2)\right]_{OPE} = \sum_{D=0,2,4,\cdots} \frac{C_D}{Q^D}\,. \qquad (2)$$

The C_D depend logarithmically on Q^2. For $D = 4$, the coefficient functions multiplying $\langle \alpha_s G^2\rangle$ and $\langle m_q \bar{q} q\rangle$ are known beyond leading order (see, e.g., [2]). For $D > 4$

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we follow standard convention and treat C_D as an effective constant. Corrections to this approximation are suppressed by additional factors of α_s and accounted for in an averaged sense if C_D is fitted to data. To fit the C_D it is convenient to employ polynomial weights $w(y) = \sum_k w_k y^k$, with $y = s/s_0$, since OPE contributions to the RHS of Eq. (1) with different D then scale differently with s_0. Such $w(y)$ yield $D \geq 6$ contributions

$$\sum_{k=2,3,\cdots} (-1)^k w_k \frac{C_{2k+2}}{s_0^k} \, . \tag{3}$$

A $w(y)$ of degree N thus generates contributions, unsuppressed by additional powers of α_s, up to $D = 2N + 2$.

"Pinched" ("unpinched") $w(s)$ are those with (without) a zero at $s = s_0$. For $s_0 \sim$ a few GeV^2, DV contributions to the RHS of Eq. (1) are known to be significant for unpinched $w(s)$ [3, 4]. Pinching in $w(s)$ significantly reduces this effect [3], compatible with the expectation that DV contributions to Π will be localized to the vicinity of the timelike point on the contour.

2. τ Decay Analyses

There are a number of recent α_s determinations based on non-strange hadronic τ decay data [4-12], all employing the 5-loop version of the $D = 0$ OPE Adler function series [5] and ALEPH [13] and/or OPAL [14] spectral data. Apart from Ref. [4], all assume DVs can be neglected, though some, either fully or partly, check this assumption for self-consistency [6, 8]. Some [4, 6, 8], but not all, attempt to fit the relevant NP OPE coefficients[1]. In the Standard Model, with $R_{V/A}$ the ratios of flavor ud V/A current-induced inclusive τ decay widths to the corresponding electronic width, $y_\tau = s/m_\tau^2$, S_{EW} a known short-distance EW correction, $w_{(00)}(y) = (1-y)^2(1+2y)$ and $w_L(y) = y(1-y)^2$, one has [1]

$$R_{V/A} = 12\pi^2 S_{EW} |V_{ud}|^2 \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} \left[w_{00}(y_\tau)\rho^{(0+1)}_{V/A}(s) - w_L(y_\tau)\rho^{(0)}_{V/A}(s) \right] , \tag{4}$$

with $\rho^{(J)}_{V/A}(s)$ the spectral function of $\Pi^{(J)}_{V/A}(s)$. $R_{V/A}$ are very accurately known experimentally. However, the degree 3 kinematic weights in Eq. (4) produce NP contributions up to $D = 8$ in the corresponding OPE representation, making $R_{V/A}$ itself insufficient to determine α_s, even if DV contributions are assumed negligible for the double-pinched weights $w_{(00)}$ and w_L.

[1]Ref. [7] assumes values for the relevant $C_{6,8}$ which are also used implicitly in Refs. [9, 10].

In Refs. [6, 13-15] an attempt was made to fit $C_{6,8}$, and hence quantify the small NP OPE contributions to $R_{V/A}$, by considering the "(km) spectral weight" analogues, $R^{(km)}_{V/A}$, of $R_{V/A}$ [16], obtained by reweighting the integrand in Eq. (4) by $(1-y_\tau)^k y_\tau^m$, and using the $km = 00, 10, 11, 12$ and 13 versions to fit the OPE parameters α_s, $\langle \alpha_s G^2 \rangle$, C_6 and C_8. This approach requires an additional implicit assumption. The weights in $R^{(km)}_{V/A}$, $km = 10, 11, 12, 13$, produce integrated, non-α_s-suppressed OPE contributions up to $D = 10, 12, 14, 16$ respectively, with each new $R^{(km)}_{V/A}$ introducing a new OPE parameter. The strategy is thus useful only if sufficiently many of the $C_{D>8}$ yield OPE contributions negligible for all weights in the analysis. Refs. [6, 13-15] implicitly assume all $C_{D>8}$ can be neglected in this sense. If this assumption fails, effects due to neglected, but non-negligible $D > 8$ contributions will be absorbed into lower D fitted parameters and yield nominal OPE contributions scaling incorrectly with s_0. The assumption can thus be tested by comparing experimental and fitted OPE versions of the $m_\tau^2 \to s_0$, $y_\tau \to y$ generalizations of the $R^{(km)}_{V/A}$. This test was performed for the V channel fits of Ref. [6] (D08) in Ref. [8] (MY08). While for the (00) case (also tested in D08) the fit deviates from experiment only below $s_0 \sim 2.5$ GeV^2, dramatic deviations are seen for the other (km) cases. Even more telling, the tests also fail for the doubly-pinched non-spectral $J = 0 + 1$ weights $w_2(y) = 1 - 2y + y^2$ and $w_3(y) = 1 - (3y/2) + (y^3/2)$ which, from Eq. (3), test separately the $D = 6$ (w_2) and $D = 8$ (w_3) parts of the NP OPE contribution to $R_{V/A}$. These problems could result from either non-negligible $D > 8$, non-negligible residual DV contributions, or both. In either case, the results of the spectral weight analyses, including those of D08, are shown to be unreliable. In assessing the reliability of other τ-based α_s results in the literature, it should be borne in mind that Refs. [5, 11, 12] did not themselves perform fits for the $C_{6,8}$ required to evaluate the $D = 6, 8$ contributions to $R_{V/A}$, but rather took these from one or more of the above spectral weight analyses.

In MY08, α_s and the C_D were fitted using the s_0-dependence of various double-pinch-weighted spectral integrals. Double-pinching was assumed sufficient to make residual DV contributions negligible, but no assumptions about the $D > 4$ OPE coefficients were made (i.e., all non-α_s-suppressed NP OPE contributions were included). The resulting OPE fits yield excellent agreement between fitted OPE and experimental spectral integrals for $s_0 \gtrsim 2$ GeV^2, not only for the weights employed in the analysis, but also for the $km = 00, 10, 11, 12, 13$ spectral weights. Fig. 1 shows the

MY08 (black curves) and D08 (red curves) V channel "fit qualities" (fitted OPE-experimental spectral integral differences scaled by the error on the latter) obtained from analyses of the same 2005 ALEPH data [13], for a range of degree ≤ 3 $w(s)$ (which yield OPE integrals depending only the $D \leq 8$ OPE parameters considered in D08). The results are obviously compatible with residual DVs being negligible and the problems in the D08 spectral weight analysis resulting from a breakdown in the assumptions about $D > 8$ contributions.

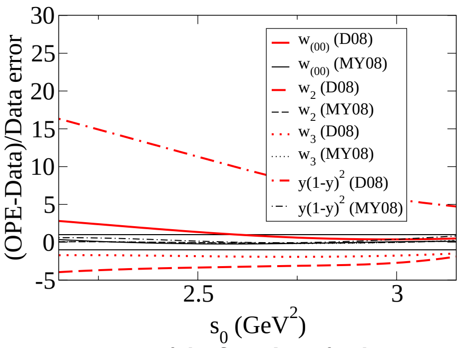

Figure 1. Comparison of the fit qualities for the OPE-only fits of Refs. [6] and [8], for a range of degree ≤ 3 weights

Two key issues remain concerning systematic errors in the MY08 approach. First, though it is consistent to neglect residual DV contributions for $s_0 \gtrsim 2$ GeV^2, a quantitative estimate of the uncertainty in α_s associated with this neglect (which would require a model for DV contributions) is absent. The second issue concerns the use of the external charmonium sum rule input for $\langle \alpha_s G^2 \rangle$ [17]. MY08 were forced to employ this input because (i) neglect of DVs was found to be inconsistent for the single-pinched, degree 1, $w(y) = 1 - y$ FESR, which one might have hoped would allow a fit of $\langle \alpha_s G^2 \rangle$ using τ data alone; (ii) doubly-pinched weights, with better suppression of DV contributions, produce OPE contributions up to at least $D = 6$; and (iii) the window of s_0 for which the neglect of DVs in doubly-pinched FESRs is self-consistent is insufficiently broad to allow simultaneous extraction of α_s, $\langle \alpha_s G^2 \rangle$ and one of the higher C_D. The use of external input is potentially problematic because of the renormalon ambiguity in the definition of $\langle \alpha_s G^2 \rangle$. This might require different values for the *effective* $\langle \alpha_s G^2 \rangle$ obtained by analyzing different correlators treated with different $D = 0$ series truncation orders. If so, the $D = 4$-induced error on α_s might be significantly larger than that generated using only the uncertainty on $\langle \alpha_s G^2 \rangle$ from the charmonium sum rule analysis. We now elaborate on, and illustrate, some of the points just made.

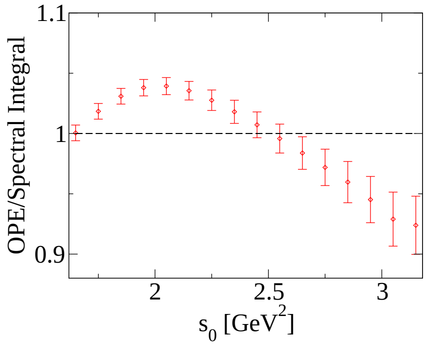

Figure 2. Duality violations for the V channel $w = 1$ FESR

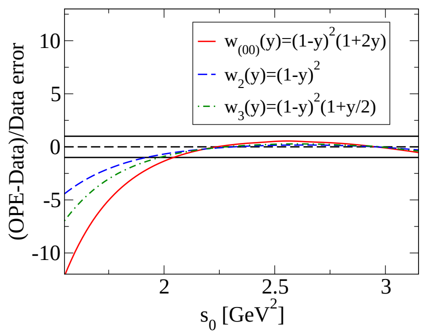

Figure 3. MY08 $w_{(00)}$, w_2 and w_3 V channel fit qualities

First, regarding DVs, it is clear that, even for $s \sim 3$ GeV^2, significant DVs are evident in the experimental spectral functions [13, 14]. Considering integrals over the spectral functions does not necessarily improve the situation, as demonstrated in Fig. 2, which shows the ratio of V channel OPE to spectral integrals for the unpinched weight $w(s) = 1$. The error bars reflect the spectral integral errors, obtained from the experimental covariance matrix. Fig. 3, which shows the fit qualities corresponding to the MY08 analysis for the $J = 0 + 1$ doubly-pinched $w_{(00)}$, w_2 and w_3 FESRs, demonstrates that, while residual DVs may be small above $s_0 \gtrsim 2$ GeV^2, they turn on rapidly below this point, making an estimate of the residual contributions in the MY08 fit window highly desirable. Finally, Fig. 4 illustrates the inability to obtain well-constrained fits to more than two OPE parameters in the limited fit window above $s_0 \sim 2$ GeV^2. The figure shows the fit qualities for the optimized w_2 V channel fits for input $\langle \alpha_s G^2 \rangle$ ranging from 3σ below to 3σ above the central charmonium sum rule value. It is obvious that all input $\langle \alpha_s G^2 \rangle$ in this

range produce essentially equally good optimized fits, and hence that the τ data alone cannot be used to successfully constrain $\langle\alpha_s G^2\rangle$. The strong anti-correlation between output α_s and input $\langle\alpha_s G^2\rangle$ means that, if one were maximally conservative, and attempted to rely on the τ data alone, sticking to the doubly-pinched weights for which residual DVs are safely small, one would be left with a contribution to the error on α_s easily three times the component quoted in MY08 associated with the charmonium sum rule error on $\langle\alpha_s G^2\rangle$. Such an expanded assessment of the $D = 4$ input uncertainty yields a contribution $\sim$ 0.0020 to the uncertainty on $\alpha_s^{n_f=5}(M_Z^2)$, already larger than the error quoted in any of the recent analyses not including DVs.

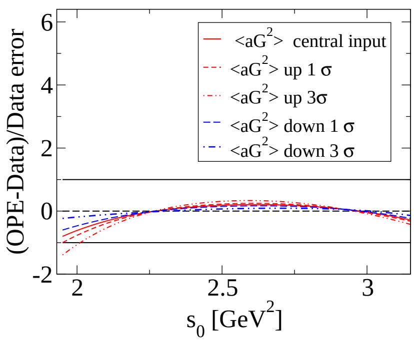

Figure 4. MY08 w_2 V channel fit qualities with input $\langle\alpha_s G^2\rangle$ from 3σ below to 3σ above the charmonium sum rule [17] central value.

To summarize the τ situation: (i) residual DVs can be plausibly neglected in the kinematically accessible region only for doubly-pinched weights, and, for these, only in the window $s_0 \gtrsim 2$ GeV^2; (ii) using τ data alone, such an s_0 window is insufficiently wide to allow the NP OPE coefficients needed for a precision determination of α_s to be fit with sufficient accuracy to achieve the precision claimed by recent analyses which neglect DV effects; (iii) this situation can be improved by extending the fit window to lower s_0 and/or including additional degree 0 and/or 1 weights (which introduce no new OPE parameters, but are, unavoidably, at most singly pinched); (iv) for either of these improvement options, residual DVs will no longer be negligible; and (v) *a reliable improved analysis thus necessarily requires use of some model to account for DV effects.* The accompanying paper (see M. Jamin, these proceedings) reports on the results of such an analysis, obtained using the DV model of Refs. [18].

3. Electroproduction-cross-section-based analyses

The spectral function, $\rho_{EM}(s)$, of the EM current scalar two-point function, $\Pi_{EM}(s)$, is related to the inclusive bare electroproduction cross-section by

$$\rho_{EM}(s) = \frac{s\,\sigma_{bare}(s)}{16\pi^3\alpha_{EM}(0)^2}, \qquad (5)$$

allowing α_s to be determined from FESRs based on inclusive electroproduction data. An advantage of this approach over that based on τ decay data is that no kinematic bound exists on the upper limit, s_0, of the spectral integral in Eq. (1). Since the integrated $D = 0$ OPE contribution to the RHS of this equation grows roughly linearly with s_0, while both integrated NP OPE and integrated DV contributions decrease with increasing s_0, the theoretical representation is more and more dominated by its perturbative, α_s-dependent contribution as s_0 increases. In addition, the $I = 1$ part of Π_{EM} is related by CVC to $\Pi_V^{(1)}$, allowing the τ data to be used to (i) estimate the small $I = 1$ higher D NP OPE and DV contributions to the RHSs of the EM FESRs and (ii) identify those s_0 for which such contributions can be safely neglected. Presently the experimental situation is complicated by (i) discrepancies between the BaBar and KLOE $\pi\pi$ cross-section data [19] and (ii) discrepancies much larger than typically associated with isospin-breaking effects between preliminary BaBar [20] and SND [21] $\pi^+\pi^-\pi^0\pi^0$ cross-sections and expectations based on experimental $\tau \to 4\pi\nu_\tau$ distributions and CVC. Some very preliminary exploratory results were presented in the conference talk, and work on this analysis is ongoing.

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